AN ASTRONOMIC MERIDIAN FOR RURAL SURVEYS *

by Prof. D.A. Wahlstrom, J.D. ** ***

INTRODUCTION

It soon becomes apparent to anyone involved in aiding people preparing to take a state land surveying registration examination that a significant number of surveying practitioners are uncomfortable and/or unfamiliar with the procedures involved when making astronomic observations. However, surveyors occasionally have to determine an astronomic direction to determine the bounds of a parcel. Consider the parcel which is described below.

> BEGINNING at an iron stake driven in the ground on the southerly bounds of Mud Pond Road, said point being 800 feet, more or less, generally easterly from the centerline of the intersection of Elm Hollow Road and Mud Pond Road; thence South 25 degrees West five chains; thence East five chains; thence North 25 degrees East to the Mud Pond Road to the point of beginning; containing acres of land.[1]

Because the tract of land was located in the midst of its parent tract, it became necessary to determine the astronomic bearing of each line.

> *** [A]nd the court would not be warranted in giving to the word “north,” ... any other than its ordinary meaning *** The word “north,” unless qualified or controlled by other words, means “due north.”[2]

> Unless other terms of a deed ... show that a different method was intended by the parties, a “due north” call should be surveyed on an astronomic basis.[3]

THEORY

Virtually every text on plane surveying covers the topic of astronomic observations. Therefore, no attempt is made in the instant paper to “re-write the book” on astronomic observations.[4] (Readers should consult the footnotes for typical references.) However, the majority of student oriented texts tend to advocate the use of the altitude method for determining the astronomic azimuth of a line.[5]

The primary advantage to using the altitude method when making solar observations is that critical timing of the solar observation is not imperative.[6] Difficulties with the method include the difficulties associated with setting both the horizontal and the vertical cross hairs tangential with the sun simultaneously and the difficulties in determining corrections for refraction.[7]

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Precise timekeeping devices are now readily available. As a result, the hour angle method of making solar observations should be considered. “[With] an accurate knowledge of the chronometer error on local time or Universal Time, the hour angle method will yield somewhat better results than that using altitudes.”[8] Another advantage is the fact that only the leading or trailing limb of the sun has to be brought tangent to the vertical cross hair. Also, no correction for refraction is required.[9]

The azimuth of the sun, using the hour angle method, can be determined by solving the following equation:

$$\tan Z = \sin t / (\cos \phi \tan \delta - \sin \phi \cos t) \qquad \textbf{E1}$$

where Z is the azimuth of the sun (**not** an azimuth in the general sense of the term, but more like a NE / SE bearing in the morning and a NW / SW bearing in the afternoon),

t is the meridian angle (the lesser of LHA or 360° - LHA),

f is the latitude of the observer, and

d is the declination of the sun.[10]

Appendix A contains the solution of an example problem using the hour angle equation.[11] The HP-41C calculator program listed in Appendix B solves the equation directly.

FIELD PROCEDURE

The field procedure for gathering the data necessary to determine the astronomic azimuth of a traverse line follows. The field procedure must be facilitated by the use of the following pieces of specialized equipment:

1. Darkening filter for the telescope eyepiece. ($35)

2. Radio Shack Timekube. ($35)

3. Quartz controlled chronograph. ($30)

Observations should be made before 9 am (10 am DST) and after 3 pm (4 pm DST).[12, 13]

1. Set up and carefully level the theodolite on an existing traverse station. *Caveat:* When the instrument is “disleveled,” the telescope does not move in a vertical plane and any horizontal angle thus measured will be adversely affected.[13A]

2. With the instrument in the face-left position, set zero, and backsight the reference mark.

3. Start the Timekube and, at the moment of the minute time-tick, start the chronograph.

4. Place the darkening filter on the telescope eyepiece.

5. Sight the sun setting the vertical cross hair so that the leading limb of the sun will “run into” the cross hair.

6. At the instant the sun’s leading limb becomes tangent with the vertical cross hair, stop the chronograph using the lap feature.

7. Record the Coordinated Universal Time (UTC) from the Timekube, the elapsed time from the chronograph, and the horizontal angle between the reference mark and the Sun.

8. Restart the chronograph using the lap feature.

9. Repeat Steps 5 through 8 for the second and third observations.

10. Change the position of the instrument to face-right.

11. Sight the sun, setting the vertical cross hair so that the trailing limb of the sun will "run away" from the cross hair.

12. At the moment the sun's trailing limb becomes tangent with the vertical cross hair, stop the chronograph using the lap feature of the chronograph.

13. Record the Coordinated Universal Time (UTC) from the Timekube, the elapsed time from the chronograph, and the horizontal angle between the reference mark and the Sun.

14. Repeat Steps 11 through 13 for the fifth and sixth observations.

15. Sight the reference mark and record the observation. (It should be 180°.)

16. Scale the latitude and longitude of the theodolite's station from a USGS quad sheet.

17. Using a current ephemeris, determine the values of the sun's declination (d) and the Equation of Time (E.O.T.) at zero hours Greenwich Civil Time for the date of the observation and for the date immediately following the observation.

18. Determine the actual time of each observation by adding the DUT1 correction to the UTC reading to get a reference UT1 for the Timekube. Then, subtract each chronograph reading from the reference UT1 to get the actual UT1 for each observation.

19. Solve for the azimuth of the line using the HP-41C calculator program outlined in Appendix B.

20. Graph the results obtained *supra* and thoroughly analyze the data. [Remember, the graph should appear as two separate, parallel lines separated by a distance equal to the semi-diameter of the sun adjusted for altitude.[14]]

21. Average the results obtained *supra* to determine the final azimuth of the line. [The average value will only be valid if a number of face-left observations are averaged with an equal number of face-right observations. No appreciable error will result if the time interval between observations is short (2 to 3 minutes).[15]]

22. Convert the final azimuth determined *supra* to the grid azimuth for the particular SPCS zone.[16]

SUMMARY

The azimuth of a traverse line, determined by following the field procedures and the computational procedures outlines *supra*, is compatible in precision with traversing measurements made for rural surveys. The primary advantages of the outlined procedure are that the field data can be collected in less than ten minutes and the azimuth can be computed in less than ten minutes.

An azimuth with an uncertainty of ± 20 seconds (95% certain) will result from three sets (3D & 3R) of observations using a Wild T-1. An azimuth with an uncertainty of ± 10 seconds (95% certain) will result from three sets (3D & 3R) of observations using a Wild T-2.[17]

SOLAR EPHEMERIS by Prof. D.A. Wahlstrom, J.D., Univ. of Houston[1]

MARCH 1995

Date (MMDD.YYYY)	G.H.A. (DDD MM SS.s)	Declination (DD MM SS.s)	E.O.T. (MM SS.s)	Semi Diam. (MM SS.s)
301.1995	176 51 44.1	- 7 50 41.3	-12 33.1	16 10.2
302.1995	176 54 35.6	- 7 27 54.4	-12 21.6	16 9.9
303.1995	176 57 34.7	- 7 5 1.0	-12 9.7	16 9.7
304.1995	177 0 41.2	- 6 42 1.7	-11 57.3	16 9.5
305.1995	177 3 54.7	- 6 18 56.8	-11 44.4	16 9.2
306.1995	177 7 15.2	- 5 55 46.7	-11 31.0	16 9.0
307.1995	177 10 42.2	- 5 32 31.8	-11 17.2	16 8.7
308.1995	177 14 15.6	- 5 9 12.6	-11 3.0	16 8.5
309.1995	177 17 55.0	- 4 45 49.3	-10 48.3	16 8.2
310.1995	177 21 40.2	- 4 22 22.4	-10 33.3	16 8.0
311.1995	177 25 30.8	- 3 58 52.3	-10 17.9	16 7.7
312.1995	177 29 26.7	- 3 35 19.3	-10 2.2	16 7.5
313.1995	177 33 27.4	- 3 11 43.8	- 9 46.2	16 7.2
314.1995	177 37 32.6	- 2 48 6.2	- 9 29.8	16 6.9
315.1995	177 41 42.0	- 2 24 26.9	- 9 13.2	16 6.7
316.1995	177 45 55.3	- 2 0 46.2	- 8 56.3	16 6.4
317.1995	177 50 12.1	- 1 37 4.4	- 8 39.2	16 6.1
318.1995	177 54 31.9	- 1 13 21.9	- 8 21.9	16 5.9
319.1995	177 58 54.6	- 0 49 39.1	- 8 4.4	16 5.6
320.1995	178 3 19.5	- 0 25 56.2	- 7 46.7	16 5.3
321.1995	178 7 46.5	- 0 2 13.6	- 7 28.9	16 5.1
322.1995	178 12 15.1	+ 0 21 28.3	- 7 11.0	16 4.8
323.1995	178 16 45.1	+ 0 45 9.2	- 6 53.0	16 4.5
324.1995	178 21 15.9	+ 1 8 48.7	- 6 34.9	16 4.2
325.1995	178 25 47.5	+ 1 32 26.5	- 6 16.8	16 4.0
326.1995	178 30 19.4	+ 1 56 2.2	- 5 58.7	16 3.7
327.1995	178 34 51.3	+ 2 19 35.3	- 5 40.6	16 3.4
328.1995	178 39 23.2	+ 2 43 5.6	- 5 22.5	16 3.1
329.1995	178 43 54.6	+ 3 6 32.7	- 5 4.4	16 2.9
330.1995	178 48 25.3	+ 3 29 56.1	- 4 46.3	16 2.6
331.1995	178 52 55.1	+ 3 53 15.6	- 4 28.3	16 2.3
332.1995	178 57 23.8	+ 4 16 30.7	- 4 10.4	16 2.0

[1]Compiled QuickBasic programs to solve solar observation problems and to generate ephemeris data are available from the author. Simply send a formatted disk and return postage.

APPENDIX A

Solar Problem

GIVEN: Time of Observation — 10:30:35 am, C.S.T., March 5, 1995
Latitude (ϕ) = 29º43’20”, Longitude (λ) = 95º20’46”
Houston, South Central Zone, Texas State Plane Coordinate System

FIND: Bearing of the Sun at time of observation.

Standard Time of Observation (from time signal plus stop watch)

at start of observation (“military” time)	10h30m35s
plus Time Zone adjustment	6h00m00s
plus Daylight Savings adjustment (as applicable)	0h00m00s
Greenwich Time of Observation	16h30m35s

Equation of Time (EOT)

at start of today	- 11m44.4s
at start of tomorrow	- 11m31.0s
at time of observation	- 11m35.2s

Declination (δ)

at start of today	- 6º18’56.8”
at start of tomorrow	- 5º55’46.7”
at time of observation	- 6º03’00.5”

Greenwich Hour Angle (GHA)

GHA = Greewich Time of Observation converted to arc plus EOT less 180º — 64º44’57”

Hour Angle

Hour Angle (t) = absolute value of GHA less West Longitude — 30º35’49”

‘Azimuth’ (Z) of the Sun

tan Z = sin t / [(cos ϕ * tan δ) - (sin ϕ * cos t)]

(A minus indicates that the angle is to be measured from South. The TIME determines in which direction to measure the angle.) — Z = S44º27’10”E

Grid Azimuth of the Sun

Grid = Geodetic - Θ + second term

Grid = 135º32’50” - 1º47’24” — NA = 133º45’26”

APPENDIX B

The calculator program listed infra (Label “HA”) can be used to determine the astronomic azimuth, the grid azimuth, the mean of the astronomic azimuth, and the mean of the grid azimuth of a traverse line by use of the hour angle method. The program is self-prompting and is activated by entry of the data followed by pressing the R/S key. All data are entered in the DD.MMSS or HH.MMSS format as appropriate.

Program “HA” returns values for the astronomic azimuth (AZM=) and the grid azimuth (GRID=) in DD.MMSS. However, the program ignores the second term when computing the grid azimuth. In order to compute the grid azimuth, it is necessary to store the angle value (in DD.dd) of the Central Meridian (C.M.) of the SPSC zone in register 00. Also, if the project is in a Lambert projection zone, change Step 121 to RCL 19, delete Step 122, and store the value of in register 19. **Values of the C.M. and “ell” can be found in SPSC Projection Tables.** SIZE should be set to 020.

Values for the declination and the Equation of Time are entered for the date in question and the next date, thereby eliminating errors associated with choosing the wrong sign for the hourly change in these values.

The mean of any number of computations can be computed at any time by pressing the LN key.

```
01   “01/18/82 V4”
02   LBL “HA”
03   FIX 4
04   CLΣ
05   “LAT”
06   PROMPT
07   HR
08   STO 03
09   “LONG”
10   PROMPT
11   HR
12   STO 02
13   “DECL”
14   PROMPT
15   HR
16   STO 05
17   “DECL +1DAY”
18   PROMPT
19   HR
20   STO 06
21   “E O T”
22   PROMPT
23   HR
24   STO 07
25   “EOT +1DAY”
26   PROMPT
27   HR
28   STO 08
29   LBL “XX”
30   “U T C”
31   PROMPT
32   HR
33   STO 01
34   “<RT”
35   PROMPT
36   HR
37   STO 04
38   RCL 08
39   RCL 07
40   -
41   24
42   /
43   RCL 01
44   *
45   RCL 07
46   +
47   RCL 01
48   +
49   15
50   *
51   RCL 02
52   -
53   180
54   -
55   ABS
56   STO 09
57   RCL 06
58   RCL 05
59   -
60   24
61   /
62   RCL 01
63   *
64   RCL 05
65   +
66   TAN
67   RCL 03
68   COS
69   /
70   RCL 03
71   SIN
72   RCL 09
73   COS
74   *
75   -
76   RCL 09
77   SIN
78   X<>Y
79   /
80   ATAN
81   X<0?
82   GTO “S”
83   GTO “N”
84   LBL “S”
85   180
86   +
87   LBL “N”
88   STO 10
89   RCL 01
90   17
91   X<=Y?
92   GTO “PM”
93   GTO “VV”
94   LBL “PM”
95   360
96   RCL 10
97   -
98   STO 10
99   LBL “VV”
100  RCL 10
101  RCL 04
102  -
103  X>0?
104  GTO “WW”
105  360
106  +
107  LBL “WW”
108  ENTER
109  Σ+
110  RDN
111  HMS
112  “AZM=“
113  ARCL X
114  AVIEW
115  STOP
116  LBL “GG”
117  HR
118  RCL 00
119  RCL 02
120  -
121  RCL 03
122  SIN
123  *
124  -
125  HMS
126  “GRID=“
127  ARCL X
128  AVIEW
129  STOP
130  GTO “XX”
131  LBL 05
132  MEAN
133  HMS
134  “MEAN=“
135  ARCL X
136  AVIEW
137  STOP
138  GTO “GG”
139  .END.
```

ENDNOTES

1. *Liber of Deeds 751* at page 588, Sullivan County Clerk's Office, New York.

2. *Currier v. Nelson*, 96 Cal. 545, 31 P. 531 (1892).

3. *Richfield Oil Corp. v. Crawford*, 39 Cal. 2d 729, 249 P.2d 600 (1952). But see *Martin v. Tucker*, 111 R.I. 179, 300 A.2d 480 (1973) (finding that "due" south was equivalent to magnetic south).

4. See R. Buckner, *Astronomic and Grid Azimuth* (sic) (1984) for an excellent, comprehensive reference on the subject.

5. See R. Davis, F. Foote, & J. Kelly, *Surveying: Theory and Practice* (5th ed. 1966) at 539 for a typical discussion regarding the determination of an azimuth by the altitude method.

6. P. Kissam, *Surveying for Civil Engineers* (1956) at 236.

7. J. Mackie, *The Elements of Astronomy for Surveyors* (8th ed. 1978) at 188.

8. *Id.*

9. *Id.*

10. M. Schmidt & W. Raynor, *Fundamentals of Surveying* (2nd ed. 1978) at 218.

11. See also *id.* at 224 for the solution of a typical example problem using Equation E1 *supra.*

12. P. McDonnell, Jr., *Selected Topics in Control Surveying* (rev. 1978) at 34.

13. See J. Mackie, *supra* note 7, at 185 for a discussion of the effect errors in the various parameters have on the determination of the final azimuth value.

13A. D. Clark, *Plane and Geodetic Surveying: Vol. 2* (6th ed. 1973) at 57.

14. See R. Brinker & P. Wolf, *Elementary Surveying* (6th ed. 1977) at 340 for a typical graph. See also F. Moffitt & H. Bouchard, *Surveying* (6th ed. 1982) at 507 for guidelines in evaluating the collected data.

15. J. Mackie, *supra* note 13, at 184.

16. R. Buckner, "Reasons and Methods for Accurate Direction in Land Surveys," *Surveying and Mapping Journal*, A.C.S.M. (Dec. 1975) at 307.

17. *Id.* See also *id.* at 312 for the number of sets of observations suggested for various accuracy requirements.

18. K. Curtis, *Meridian Determination by Solar and Polaris Observation* (sic), Indiana Society of Professional Land Surveyors (1975) at 46.